Rigid Body

Tasks

1. Two balls of masses m and $2m$ are placed on two thin rods of negligible mass according to Fig. The mass $m = 10$ g, the length of the rods is 40 cm. Determine the moments of inertia of the two rods with balls with respect to the axis perpendicular to them and passing through their ends. result

2. What is the energy of a circular disc of mass 8 kg and radius 25 cm if it makes 500 rotations in one minute? The moment of inertia of the circular disc with respect to the axis of rotation passing perpendicularly through its centre is $J = \frac{1}{2}mR^2$. result

3. What is the velocity of a ball rolling down an inclined plane from a height of 1 m? The moment of inertia of a homogeneous sphere of mass m and radius R with respect to an axis passing through its centre is $J = \frac{2}{5}mR^2$. We do not consider friction. result

4. How far would a wheel with a mass of 20 kg and a radius of 34 cm roll if it were released from the axis of the car at a speed of 72 km/h. The moment of inertia of the wheel is $1\,\mathrm{kg\,m^2}$, the magnitude of the drag force is 4% of the gravitational force acting on the wheel. result

5. Two identical balls of mass m are placed on a thin rigid rod of negligible mass 75 cm in length: one at the end of the rod, the other in the centre of the rod m (Fig.). The rod can rotate around a horizontal axis passing through point O perpendicular to the drawing. What velocity must be given to the lower end of the rod to cause it to deflect from vertical to horizontal? result

6. Three balls are placed on a thin, rigid horizontal rod of negligible mass rotating about a horizontal axis passing through the point O perpendicular to the drawing (Fig.). Two balls of mass m are placed on the left at distances ℓ and $\ell/2$ from the axis of rotation, one ball of mass $2m$ is placed on the right at a distance $\ell/2$ from the axis of rotation. The rod, which initially occupies a horizontal position, is released so that it begins to rotate about the axis of passing through point O. Determine the magnitude of the velocity of the middle ball as the rod passes through the vertical position. Solve the problem first in general and then for values of $m = 0.01$ kg, $l = 0.7$ m, $g = 10\,\mathrm{m\,s^{-2}}$. result

Results

1. $J_1 = \frac{9}{4}m\ell^2 = 3.6\,\mathrm{g\,m^2}$; $\quad J_2 = \frac{3}{2}m\ell^2 = 2.4\,\mathrm{g\,m^2}$. task

2. $E_\mathrm{k} = \dfrac{\pi^2 mR^2n^2}{t^2} \doteq \dfrac{3.14^2 \cdot 8 \cdot 0.25^2 \cdot 500^2}{60^2}\,\mathrm{J} \doteq 340\,\mathrm{J}$. task

3. $v = \sqrt{\frac{2}{5}hg} \doteq 3.8\,\mathrm{m\,s^{-1}}$. task

4. $\ell = \dfrac{v^2(mR^2 + J)}{2kmgR^2} = \dfrac{20^2(20 \cdot 0.34^2 + 1)}{2 \cdot 0.04 \cdot 20 \cdot 10 \cdot 0.34^2}\,\mathrm{m} \doteq 730\,\mathrm{m}$. task

5. $v = \sqrt{\dfrac{12}{5}\ell g} = \sqrt{\dfrac{12 \cdot 0.75 \cdot 9.81}{5}}\,\mathrm{m\,s^{-1}} \doteq 4.2\,\mathrm{m\,s^{-1}}$. task

6. $v = \sqrt{\dfrac{\ell g}{7}} = \sqrt{\dfrac{0.7 \cdot 10}{7}}\,\mathrm{m\,s^{-1}} \doteq 1\,\mathrm{m\,s^{-1}}$. task

Gravity

Tasks

1. Find the dimension of the gravitational constant as a unit of the SI system. result

2. The ratio of the radius of Mars and Earth is 0.53, the ratio of their masses is 0.11. Determine how many times the gravitational force on a body on the surface of Earth is greater than that on the surface of Mars. result

3. Determine the gravitational acceleration on the surface of Venus if the mean density of the substances that make up the planet Venus is $4\,900\ \mathrm{kg\,m^{-3}}$ and its radius is $6\,200$ km. The gravitational constant is $6.67 \cdot 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ (more about this constant here [5]). result

4. Determine the gravitational force acting on a body of mass 16 kg if it is above the surface of the Earth at a height equal to 1/3 of the Earth's radius. The acceleration due to gravity at the surface of the Earth is approximately $10\ \mathrm{m\,s^{-2}}$. result

5. Determine the height to which the body must be raised above the surface of the Earth, for the gravitational force acting on the body to be reduced by a factor of two. The radius of the Earth is approximately $6\,400$ km. result

6. Calculate orbital velocity for Earth for low orbit. What is the orbital period? result

7. The body moves in the central gravitational field in a circular orbit around the centre of symmetry. Find the relationship between the radius of the path and its velocity. result

8. The Earth moves around the Sun in a circle with a radius of approximately $1.5 \cdot 10^8$ km at a speed of $30\ \mathrm{km\,h^{-1}}$. Determine the mass of the Sun. result

9. A satellite moves around the Earth in a circle whose radius is twice, than the radius of the Earth. Determine the speed at which this satellite is moving, if the first cosmic velocity at the Earth's surface is $8\ \mathrm{km\,s^{-1}}$. result

10. Calculate the orbital period of the first artificial satellite of the Earth, Sputnik 1, if you know that the distance of the Moon from the centre of the Earth is 384 400 km, the orbital period of the Moon is 27.33 days, the radius of the Earth is 6 378 km, and the height of the satellite above the surface of the Earth is 900 km. *The Sputnik 1 satellite was sent on an orbital trajectory on October 4, 1957, and ceased to exist in January 1958.* result

11. Halley's Comet, which is moving along an elliptical trajectory, is getting to a minimum distance of 0.6 AU from the Sun at perihelion. The perihelion of Halley's of the comet is 76 years. Determine what is the greatest distance from the Sun it will reach.
More details on Haley's Comet on Wikipedia. result

12. The diameter of the image of the Sun taken at the time of the aphelion is 600 pixels. In an image taken at the time of perihelion under the same conditions, the diameter is 620 pixels. Find the relative eccentricity of the Earth's orbit around the sun. result

13. You want to draw the Earth's orbit around the Sun on a board at a scale of 1 AU $\leftrightarrow$ 1 m.
    a) What will be the eccentricity e?
    b) What will be the distance between perihelion and aphelion?
    c) How big will the minor axis be? Hint: use approximation $\sqrt{1+\gamma} \approx 1 + \frac{1}{2}\gamma$ for $\gamma \ll 1$.

result

Results

1. $[G] = \dfrac{[F][r^2]}{[m^2]} = \mathrm{N\,m^2\,kg^{-2}} = \dfrac{\mathrm{kg\,m}}{\mathrm{s^2}}\mathrm{m^2\,kg^{-2}} = \mathrm{kg^3\,m^3\,s^{-2}}.$ task

2. $\dfrac{F_{\text{Earth}}}{F_{\text{Mars}}} = \dfrac{m_{\text{Earth}}}{m_{\text{Mars}}}\left(\dfrac{r_{\text{Mars}}}{r_{\text{Earth}}}\right)^2 = \dfrac{1}{0.11}\cdot 0.53^2 \doteq 2.6.$ task

3. $a_{\text{Venus}} = \dfrac{4}{3}G\pi\rho R \doteq \dfrac{4}{3}6.67\cdot 10^{-11}\cdot 3.14\cdot 4\,900\cdot 6.2\cdot 10^6\ \mathrm{m\,s^{-2}}.$ task

4. $F = G\dfrac{m\cdot m_{\text{Earth}}}{\left(\frac{4}{3}R\right)^2} = m\cdot \underbrace{G\dfrac{m_{\text{Earth}}}{R^2}}_{g}\cdot\left(\dfrac{3}{4}\right)^2 = \dfrac{9}{16}mg \doteq \dfrac{9}{16}\cdot 16\cdot 10\ \mathrm{N} = 90\ \mathrm{N}.$ task

5. $h = R_{\text{Earth}}(\sqrt{2}-1) \doteq 6.4\cdot 10^6(1.4-1)\ \mathrm{m} \doteq 2.65\cdot 10^6\ \mathrm{m}.$ task

6. $v = \sqrt{\dfrac{Gm}{R}} = \sqrt{\dfrac{Gm_{\text{Earth}}}{R^2}R} = \sqrt{gR} \doteq \sqrt{10\cdot 6.4\cdot 10^6}\ \mathrm{m\,s^{-1}} = 8\cdot 10^3\ \mathrm{m\,s^{-1}} = 8\ \mathrm{km\,s^{-1}}$;
   note that with $R \doteq 6.4\cdot 10^6$ m and $g \doteq 10\ \mathrm{m\,s^{-2}}$ you can easily calculate the square root;
   $T = \dfrac{2\pi R}{v} = \dfrac{2\cdot 3.14\cdot 6.4\cdot 10^6}{8\,000} \doteq 5\,000\ \mathrm{s} \doteq 84\ \mathrm{min}$; see on Wolfram Alpha. task

7. $Gmm_{\text{c}} = rv^2$ task

8. $m_{\text{Sun}} = \dfrac{rv^2}{G} = \dfrac{1.5\cdot 10^{11}\cdot(3\cdot 10^4)^2}{6.67\cdot 10^{-11}}\ \mathrm{kg} \doteq 2\cdot 10^{30}\ \mathrm{kg}.$ task

9. $v_2 = \dfrac{v_1}{\sqrt{2}} \doteq \dfrac{8\,000}{1.41}\ \mathrm{m\,s^{-1}} \doteq 5.7\ \mathrm{km\,s^{-1}}.$ task

10. $T = \sqrt{\dfrac{r^3}{r_{\text{M}}^3}}T_{\text{M}} = \sqrt{\dfrac{7\,278^3}{384\,400^3}}655.9\ \mathrm{h} \doteq 1.709\ \mathrm{h} = 102\ \mathrm{min}$ (1:42 in h:mm format). task

11. $a_{\text{H}} = r_{\text{mean}} = a_{\text{E}}\sqrt[3]{\dfrac{T_{\text{H}}^2}{T_{\text{E}}^2}} = (1\ \mathrm{AU})\sqrt[3]{\dfrac{(76\ \mathrm{y})^2}{(1\ \mathrm{y})^2}} \doteq 17{,}9\ \mathrm{AU}$;
    $r_{\text{peri}} + r_{\text{aphe}} = 2r_{\text{mean}} \to r_{\text{aphe}} = 2r_{\text{mean}} - r_{\text{peri}} = (2\cdot 17{,}9 - 0.6)\ \mathrm{AU} = 35.2\ \mathrm{AU}.$ task

12. $\varepsilon = \dfrac{r_{\text{aphe}} - r_{\text{peri}}}{r_{\text{aphe}} + r_{\text{peri}}} = \dfrac{D_{\text{aphe}} - D_{\text{peri}}}{D_{\text{aphe}} + D_{\text{peri}}} = \dfrac{620-600}{620+600} \approx \dfrac{1}{60}.$ task

13. a) $e = \epsilon a \approx 1/60\ \mathrm{m} \doteq 0.0167\ \mathrm{m} = 1.67\ \mathrm{cm}$;
    b) $r_{\text{aphe}} - r_{\text{peri}} = 2e \doteq 3.34\ \mathrm{cm}$;
    c) $b = a\sqrt{1-\dfrac{1}{60^2}} \approx 1\left(1-\dfrac{1}{7\,200}\right) \doteq 0.99986\ \mathrm{m} = 1\ \mathrm{m} - 0.14\ \mathrm{mm}$;
    note that the shape of the trajectory drawn on the board is unrecognizable from circle. task

Quantities

- $\mathbf{M}$ – ($\mathbf{T}$, $\boldsymbol{\tau}$), *turning moment*, (or *torque*) [3], $\mathbf{M} = \mathbf{r} \times \mathbf{F}$ (N m)
- M – turning moment relative to the fixed axis, moment of twisting force, $M = \ell F_{\perp}$, (N n)
- ω – *angular velocity* to the fixed axis, $\omega = 2\pi f$ (s^{-1}, rad^{-1})
- f – *frequency* (s^{-1}, Hz)
- ε – (α), *angular acceleration* to the fixed axis, $\varepsilon = \dot{\omega}$ (s^{-2}, rad^{-2})
- J – (I), *moment of inertia* relative to the fixed axis, [4], $J = mr^2$ for mass point, ($\mathrm{kg\,m}^2$)
- $\mathbf{p}$ – *momentum*, $\mathbf{p} = m\mathbf{v}$ for a mass point ($\mathrm{kg\,m\,s}^{-1}$)
- $\mathbf{b}$ – *angular momentum*, $\mathbf{b} = \mathbf{r} \times \mathbf{p}$ for a mass point, $\mathbf{L}$ in quantum mechanics, ($\mathrm{kg\,m\,s}^{-1}$)

Literature

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https://en.wikipedia.org/wiki/Angular_frequency

[2] Moment (physics), Wikipedia
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[3] Torque, Wikipedia
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[4] Moment of inertia, Wikipedia
https://en.wikipedia.org/wiki/Moment_of_inertia

[5] Gravitational constant, Wikipedia
https://en.wikipedia.org/wiki/Gravitational_constant

[6] Math symbols at Math Vault
https://mathvault.ca/hub/higher-math/math-symbols/