Rigid Body

Tasks

1. Two balls of masses m and $2m$ are placed on two thin rods of negligible mass according to Fig. The mass $m = 10$ g, the length of the rods is 40 cm. Determine the moments of inertia of the two rods with balls with respect to the axis perpendicular to them and passing through their ends. result

2. What is the energy of a circular disc of mass 8 kg and radius 25 cm if it makes 500 rotations in one minute? The moment of inertia of the circular disc with respect to the axis of rotation passing perpendicularly through its centre is $J = \frac{1}{2}mR^2$. result

3. What is the velocity of a ball rolling down an inclined plane from a height of 1 m? The moment of inertia of a homogeneous sphere of mass m and radius R with respect to an axis passing through its centre is $J = \frac{2}{5}mR^2$. We do not consider friction. result

4. How far would a wheel with a mass of 20 kg and a radius of 34 cm roll if it were released from the axis of the car at a speed of 72 km/h. The moment of inertia of the wheel is $1\,\mathrm{kg\,m^2}$, the magnitude of the drag force is 4% of the gravitational force acting on the wheel. result

5. Two identical balls of mass m are placed on a thin rigid rod of negligible mass 75 cm in length: one at the end of the rod, the other in the centre of the rod m (Fig.). The rod can rotate around a horizontal axis passing through point O perpendicular to the drawing. What velocity must be given to the lower end of the rod to cause it to deflect from vertical to horizontal? result

6. Three balls are placed on a thin, rigid horizontal rod of negligible mass rotating about a horizontal axis passing through the point O perpendicular to the drawing (Fig.). Two balls of mass m are placed on the left at distances ℓ and $\ell/2$ from the axis of rotation, one ball of mass $2m$ is placed on the right at a distance $\ell/2$ from the axis of rotation. The rod, which initially occupies a horizontal position, is released so that it begins to rotate about the axis of passing through point O. Determine the magnitude of the velocity of the middle ball as the rod passes through the vertical position. Solve the problem first in general and then for values of $m = 0.01$ kg, $l = 0.7$ m, $g = 10\,\mathrm{m\,s^{-2}}$. result

Results

1. $J_1 = \frac{9}{4}m\ell^2 = 3.6\,\mathrm{g\,m^2}; \quad J_2 = \frac{3}{2}m\ell^2 = 2.4\,\mathrm{g\,m^2}.$ task

2. $E_\mathrm{k} = \dfrac{\pi^2 mR^2n^2}{t^2} \doteq \dfrac{3.14^2 \cdot 8 \cdot 0.25^2 \cdot 500^2}{60^2}\,\mathrm{J} \doteq 340\,\mathrm{J}.$ task

3. $v = \sqrt{\frac{2}{5}hg} \doteq 3.8\,\mathrm{m\,s^{-1}}.$ task

4. $\ell = \dfrac{v^2(mR^2+J)}{2kmgR^2} = \dfrac{20^2(20\cdot 0.34^2+1)}{2\cdot 0.04\cdot 20\cdot 10\cdot 0.34^2}\,\mathrm{m} \doteq 730\,\mathrm{m}.$ task

5. $v = \sqrt{\frac{12}{5}\ell g} = \sqrt{\dfrac{12\cdot 0.75\cdot 9.81}{5}}\,\mathrm{m\,s^{-1}} \doteq 4.2\,\mathrm{m\,s^{-1}}.$ task

6. $v = \sqrt{\dfrac{\ell g}{7}} = \sqrt{\dfrac{0.7\cdot 10}{7}}\,\mathrm{m\,s^{-1}} \doteq 1\,\mathrm{m\,s^{-1}}.$ task

Gravity

Tasks

1. Find the dimension of the gravitational constant as a unit of the SI system. result

2. The ratio of the radius of Mars and Earth is 0.53, the ratio of their masses is 0.11. Determine how many times the gravitational force on a body on the surface of Earth is greater than that on the surface of Mars. result

3. Determine the gravitational acceleration on the surface of Venus if the mean density of the substances that make up the planet Venus is $4\,900\,\mathrm{kg\,m^{-3}}$ and its radius is $6\,200\,\mathrm{km}$. The gravitational constant is $6.67 \cdot 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$ (more about this constant here [5]). result

4. Determine the gravitational force acting on a body of mass $16\,\mathrm{kg}$ if it is above the surface of the Earth at a height equal to $1/3$ of the Earth's radius. The acceleration due to gravity at the surface of the Earth is approximately $10\,\mathrm{m\,s^{-2}}$. result

5. Determine the height to which the body must be raised above the surface of the Earth, for the gravitational force acting on the body to be reduced by a factor of two. The radius of the Earth is approximately $6\,400\,\mathrm{km}$. result

6. Calculate orbital velocity for Earth for low orbit. What is the orbital period? result

Results

1. $[G] = \mathrm{N\,m^2\,kg^{-2}}$. task
2. $\dfrac{F_{\text{Earth}}}{F_{\text{Mars}}} = $. task
3. $a_{\text{Venus}} = $. task
4. $F = $. task
5. $h = $. task
6. $v =; T = $. task

Quantities

- $\mathbf{M}$ – ($\mathbf{T}$, $\boldsymbol{\tau}$), *turning moment*, (or *torque*) [3], $\mathbf{M} = \mathbf{r} \times \mathbf{F}$ (N m)
- M – turning moment relative to the fixed axis, moment of twisting force, $M = \ell F_{\perp}$, (N n)
- ω – *angular velocity* to the fixed axis, $\omega = 2\pi f$ (s^{-1}, rad^{-1})
- f – *frequency* (s^{-1}, Hz)
- ε – (α), *angular acceleration* to the fixed axis, $\varepsilon = \dot{\omega}$ (s^{-2}, rad^{-2})
- J – (I), *moment of inertia* relative to the fixed axis, [4], $J = mr^2$ for mass point, ($\mathrm{kg\,m}^2$)
- $\mathbf{p}$ – *momentum*, $\mathbf{p} = m\mathbf{v}$ for a mass point ($\mathrm{kg\,m\,s}^{-1}$)
- $\mathbf{b}$ – *angular momentum*, $\mathbf{b} = \mathbf{r} \times \mathbf{p}$ for a mass point, $\mathbf{L}$ in quantum mechanics, ($\mathrm{kg\,m\,s}^{-1}$)

Literature

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https://en.wikipedia.org/wiki/Angular_frequency

[2] Moment (physics), Wikipedia
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[3] Torque, Wikipedia
https://en.wikipedia.org/wiki/Torque

[4] Moment of inertia, Wikipedia
https://en.wikipedia.org/wiki/Moment_of_inertia

[5] Gravitational constant, Wikipedia
https://en.wikipedia.org/wiki/Gravitational_constant

[6] Math symbols at Math Vault
https://mathvault.ca/hub/higher-math/math-symbols/